Example 4h: Predictor-Corrector Time Integration

This example problem demonstrates the use of the predictor-corrector time integration method within MAC/GMC 4.0, which employs a self-adaptive time stepping scheme. This method is based on a fourth-order Runge-Kutta starter, an Adams-Bashforth four-step predictor, and an Adams-Moulton four-step corrector. Instead of specifying an explicit time step size (as in the forward Euler time integration method), the user specifies an acceptable level of error between the predicted and corrected values (for each local and global variable). The predictor-corrector integration algorithm then automatically adjusts the time step size in order to take the largest possible time step while ensuring that the error remains below the error tolerance. For additional information see Section 7.2 of the Theory Manual. The present problem considers a unidirectional 0.35 fiber volume fraction SiC/Ti-21S composite as represented by the 7×7 circular fiber cross-section approximation RUC architecture. A stress-free cool-down is first applied to incorporate residual stresses (see Example 4b), followed by cyclic mechanical loading.

MAC/GMC Input File: **example_4h.mac**

```
MAC/GMC 4.0 Example 4h - Predictor-corrector time integration method
*CONSTITUENTS
  NMATS=2
  M=1 CMOD=6 MATID=E
  M=2 CMOD=4 MATID=A
*RUC
  MOD=2 ARCHID=6 VF=0.35 R=1. F=1 M=2
*MECH
  LOP=2 REFTIME=57600.
  NPT=8 TI=0.,57600.,57630.,57650.,57690.,57710.,57770.,57790. &
        MAG=0.,0.,0.015,0.,0.02,0.,0.03,0. MODE=2,1,2,1,2,1,2
*THERM
  NPT=3 TI=0.,57600.,57790. TEMP=900.,23.,23.
*SOLVER
  METHOD=2 ISTM=0.001 ISTT=0.1 ERR=0.001 MINSTEP=0.00001
*PRINT
  NPL=3
*XYPLOT
  FREQ=5
  MACRO=3
   NAME=example_4h     X=2   Y=8
   NAME=example_4h_th1 X=100 Y=1
   NAME=example_4h_th2 X=100 Y=2
  MICRO=0
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (***CONSTITUENTS**) [KM_2]:
   Number of materials: 2   (NMATS=2)

Materials: SiC fiber (MATID=E)
Ti-21S (MATID=A)

Constitutive models: SiC fiber: linearly elastic (CMOD=6)
Ti-21S matrix: Isotropic GVIPS (CMOD=4)

3) Analysis type (***RUC**) → Repeating Unit Cell Analysis [KM_3]:
Analysis model: Doubly periodic GMC (MOD=2)
RUC architecture: 7×7 circle approx., rect. pack (ARCHID=6)
Fiber volume fraction: 0.35 (VF=0.35)
RUC aspect ratio: 1. (square pack) (R=1.)
Material assignment: SiC fiber (F=1)
Ti-21S matrix (M=2)

4) Loading:
a) Mechanical (***MECH**) [KM_4]:
Loading option: 2 (LOP=2)
Strain reference time: 57600. sec. (REFTIME=57600.)
Number of points: 8 (NPT=8)

Times (TI=) (sec.)	0.	57600.	57630.	57650.	57690.	57710.	57770.	57790.
Magnitudes (MAG=)	0.	0	0.015	0.0	0.02	0.	0.03	0.
Control (MODE=)	stress	strain	stress	strain	stress	strain	stress	

In this example, the mechanical loading is applied in strain control and unloaded in stress control. This allows the stress to be precisely controlled so that it returns to zero after each cycle.

b) Thermal (***THERM**) [KM_4]:
Number of points: 3 (NPT=3)
Time points: 0., 57600., 57790. sec. (TI=0.,57600.,57790.)
Temperature points: 900., 23., 23. (TEMP=900.,23.,23.)

c) Time integration (***SOLVER**) [KM_4]:
Time integration method: Predictor-corrector (METHOD=2)
Initial mech. time step: 0.001 sec. (ISTM=0.001)
Initial thermal time step: 0.1 sec. (ISTT=0.1)
Error tolerance: 0.001 (ERR=0.001)
Minimum time step size: 0.00001 sec. (MINSTEP=0.00001)

The initial mechanical (ISTM) and thermal (ISTT) time steps are used to start the time integration with the fourth-order Runge-Kutta method. The error tolerance (ERR) is maximum allowable fractional difference between a predicted and corrected value for any local or global variable. The minimum time step size (MINSTEP) limits how small the time step can become, regardless of the predicted and corrected variable values. As the results of this example indicate, the above values for these terms work well in the present case, but they can be case dependent. Users are thus encouraged to employ values for the terms based on experience with their own particular problems. For additional information of the predictor-corrector, see the MAC/GMC 4.0 Keywords Manual Section 4 and the Theory Manual Section 7.2.

5) Damage and Failure: None

6) Output:

a) Output file print level (**`*PRINT`**) [KM_6]:

Print level:	3	(`NPL=3`)

b) x-y plots (**`*XYPLOT`**) [KM_6]:

Frequency:	5	(`FREQ=5`)
Number of macro plots:	3	(`MACRO=3`)
Macro plot name:	example_4h	(`NAME=example_4h`)
	example_4h_th1	(`NAME=example_4h_th1`)
	example_4h_th2	(`NAME=example_4h_th2`)
Macro plot x-y quantities:	ε_{22}, σ_{22}	(`X=2 Y=8`)
	temperature, ε_{11}	(`X=100 Y=1`)
	temperature, ε_{22}	(`X=100 Y=2`)
Number of micro plots:	0	(`MICRO=0`)

7) End of file keyword: (**`*END`**)

Results

Figure 4.20 shows the longitudinal and transverse strain induced in the SiC/Ti-21S composite as it is cooled from 900 °C to room temperature. The predictor corrector begins with a small initial time step, which soon increases as the temperature is dropping. At approximately 700 °C, the composite begins to experience inelastic deformation, causing the time step to become very small. At lower temperatures, the inelastic strain rate becomes smaller, and the time step is increased.

Figure 4.21 shows the subsequent transverse cyclic tensile response of the composite. Again, as the mechanical loading begins, the time step is small. The composite behaves as initially elastic, thus the time step increases rapidly. As the composite begins to yield, the time step decreases, and subsequently increases several times. This is due to yielding of the individual subcells within the composite. Once a given subcell yields and begins to flow at a predictable rate, the predictor-corrector can increase the time step until another subcell starts to yield. The unloading and reloading behavior of the composite is mainly elastic in nature, so the time step increases for these segments of the stress-strain response. When the composite begins to yield for each cycle, the time step decreases once more.

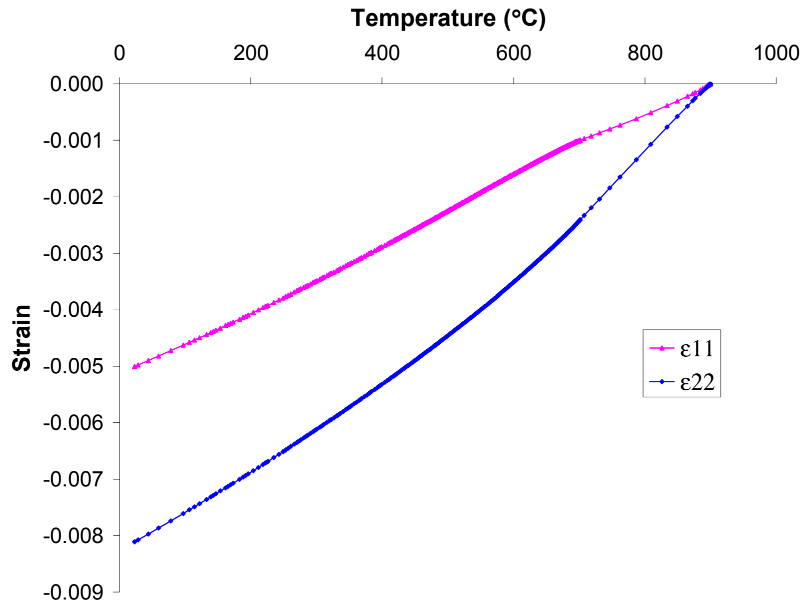

Figure 4.20 Example 4h: Stress-free cool-down response of 0.35 fiber volume fraction SiC/Ti-21S generated using the predictor-corrector time integration scheme.

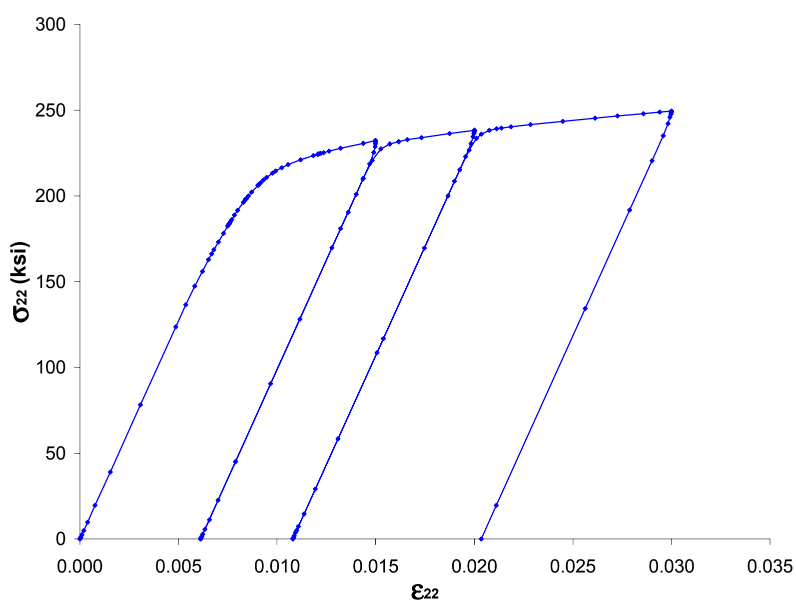

Figure 4.21 Example 4h: Cyclic transverse tensile response of 0.35 fiber volume fraction SiC/Ti-21S at room temperature generated using the predictor-corrector time integration scheme.